LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2018**

16/17PST3MC02/ST 3816/ST 3812 – STOCHASTIC PROCESSES

Date: 27-10-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Section A

Answer all the questions **(10 x 2 = 20 Marks)**

1. Define state space and index parameter of a stochastic process.
2. When a stochastic process is called a point process?
3. Write any two properties of the period of a state.
4. State Abel's lemma.
5. Write the postulates of a pure birth process.
6. Define a counting process.
7. Define supermartingale for a stochastic process.
8. Cite an example for a branching process.
9. Define a covariance stationary process.
10. Write a note on discrete renewal equation.

Section B

Answer any five questions **(5 x 8= 40 Marks)**

11. Explain the following:
    (i) Process with stationary independent increments.
    (ii) Martingales. (4 + 4) marks

12. If a process $\{X_t , t \in T\}$ where T = $[0,\infty]$ or T=(0,1,2,…) has stationary independent increments and has a finite mean , show that $E[X_t] = m_0 + m_1 t$ where $m_0 = E[X_0]$ and $m_1= E[X_1] - m_0$.
13. Show that one- dimensional random walk is recurrent.
14. State and prove the theorem used to find the stationary probability distribution of the Markov chain.
15. Discuss the limiting behavior of $P_{ij}{}^{n}$ when i is transient j is recurrent.
16. Under the condition that X(0) = N = 1 , determine the mean and variance of the Yule process.
17. Show that the variance of the sum of the independently and identically distributed random variables with finite second moment is a martingale.
18. Establish the generating function relations for branching process.

Section C

Answer any two questions **(2 x 20 = 40 marks)**

19.(a) Prove that a state i is recurrent if and only if $\sum_{i=1}^{\infty} {P_{ii}}^{n} = \infty$. .

(b) Prove that the three - dimensional random walk is transient. (5 +15).

20. (a) Derive $P_n(t)$ for the Poisson process.

(b) Derive backward and forward Kolmogorov differential equations of birth and death processes. (10 +10)

21. Establish the generating function relations for branching processes and hence find mean and variance.

22. Explain the following:

(i) a stationary process on the circle

(ii) Stationary Markov chains

(iii) Schwartz ' s inequality

(iv) Uniqueness of mean square limit

(v) Cauchy criterion for convergence (5 x 4 = 20)
